Code No. F-15205/BL

FACULTY OF SCIENCE

B.A./B.Sc. (CBCS) III – Semester (Backlog) Examination, May/June 2024

Subject : Mathematics
Paper – III : Real Analysis

Time: 3 Hours **Max. Marks: 80**

PART – A

Note : Answer any Eight questions. **(8x4=32 Marks)**

1. Find the limit the sequence $S_n = \frac{2n+3}{3n+4}$ and show that it is unique.
2. Show that the sequence $S_n = \frac{\cos n\pi}{3}$ is not monotonic.
3. Prove that every convergent sequence is a Cauchy sequence.
4. If $f: A \to B$ and $g: B \to C$ are mappings such that f is continuous at x_0 and g is continuous at $f(x_0)$, then prove that gof is continuous at x_0.
5. Explain the properties of continuous functions.
6. Prove that, if f is continuous on $[a,b]$ then it is uniformity continuous on $[a,b]$.
7. If f and g are two functions such that f and g are derivable at $a \in R$, then prove that $f+g$ and fg are derivable at $a \in R$.
8. Using mean value theorem, find the value of c for the function $f(x) = lx^2 + mx + n$ in the interval $[a,b]$
9. Find the Taylor's series expansion of $f(x) = \log(1+x)$.
10. Give an example to show that every bounded function is not Riemann integrable.
11. Prove that every monotonic function is Riemann integrable.
12. If f and g are integrable on $[a,b]$ and if $f(x) \le g(x)$ for $x \in [a,b]$, then prove that $\int_a^b f \le \int_a^b g$.

PART – B

Note : Answer all the questions. **(4x12=48 Marks)**

13. (a) (i) Prove that convergent sequences are bounded.
(ii) If the sequence (S_n) converges then prove that every subsequence converges to the same limit.

OR

(b) (i) State and prove Ratio test.
(ii) Prove that $\sum \frac{1}{n^p}$ converges if and only if p > 1.

14. (a) State and prove intermediate value theorem.

OR

(b) If f is uniform continuous on $[a, b]$ and S_n is Cauchy in $[a, b]$ then prove that $f(S_n)$ is Cauchy sequence in f([a, b]).

15. (a) State and prove Rolle's theorem.

OR

(b) State and prove Taylor's theorem.

16. (a) Prove that a bounded function f on [a, b] is Riemann integrable if and only if it is (Darboux) integrable, in which case the values of the integrals agree.

OR

(b) State and prove fundamental theorem of calculus – II.

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Code No: F–15205

FACULTY OF SCIENCE

B.A./B. Sc. (CBCS) III – Semester Examination, December 2023/January 2024

Subject: Mathematics
Paper – III: Real Analysis

Time: 3 Hours Max. Marks: 80

PART – A

Note: Answer any eight questions. (8 x 4 = 32 Marks)

1. Show that $\lim_{n\to\infty}\frac{1}{n^2}=0$.
2. Define bounded sequence and give an example.
3. Determine the nature of the series $\sum_{n=1}^{\infty}\frac{2n}{n^3+6}$.
4. Let $A\subseteq R$. If $f:A\to R$ is a continuous function on A, Then show that $|f|$ is also continuous A.
5. State intermediate value theorem.
6. If $f:R\to R$ is defined as $f(x)=\begin{cases} x\sin\frac{1}{x} & , \quad x\neq 0 \\ 0 & , \quad x=0 \end{cases}$.

   then show that f is continuous on R.
7. State Rolle's Theorem.
8. Using mean value theorem, show that $|\sin x-\sin y|\leq|x-y|$ for all $x, y\in R$.
9. Evaluate $\lim_{x\to\infty}\left(1-\frac{3}{x}\right)^x$.
10. If the function $f:[0,b]\to R$ is defined as $f(x)=x^2$, then find $L(f,P)$ where $P=\left\{0,\frac{b}{n},\frac{2b}{n},...,b\right\}$ is a partition of $[0,b]$.
11. If $f:[a,b]\to R$ is Riemann integrable on $[a,b]$ and $c\in R$, then show that $\int_a^b c\, f=c\int_a^b f$.
12. Evaluate $\lim_{h\to 0}\frac{1}{h}\int_5^{5+h}e^{t^2}\,dt$.

PART – B

Note: Answer all the questions. **(4 x 12 = 48 Marks)**

13. (a) If (s_n) converges to S and (t_n) converges to t then show that $(s_n + t_n)$ converges to $s+t$.

(OR)

(b) Determine the nature of the following series.

(i) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ (ii) $\sum_{n=1}^{\infty} \frac{n^3}{3^n}$ (iii) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{3n+5}$

14. (a) Let g be a strictly increasing function on an interval J such that $g(J)$ is an interval I. then show that g is continuous on J.

(OR)

(b) (i) Prove that $x\,2^x = 1$ for some $x \in (0,1)$.

(ii) Find the maximum value of $f(x) = x^3 - 6x^2 + 9x + 1$ on $[0, 5)$.

15. (a) State and prove Mean value theorem.

(OR)

(b) State and prove Taylor's theorem.

16. (a) If $f:[a,b] \to R$ is a monotonic function, then show that f is Riemann integrable.

(OR)

(b) (i) Evaluate $\int_0^1 x\sqrt{1-x^2}\,dx$.

(ii) Evaluate $\int_0^{\frac{1}{2}} Sin^{-1}x\,dx$.

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OU - 1110

Code No: E-10215

FACULTY OF SCIENCE

B.Sc. (CBCS) III - Semester Examination, December 2022 / January 2023

Subject: Mathematics

Paper – III: Real Analysis

Time: 3 Hours **Max. Marks: 80**

PART – A

Note: Answer any eight questions. **(8 x 4 = 32 Marks)**

1. Define Limit of Sequence and prove that the limit of sequence is unique.
2. Prove that all bounded monotone sequences converge.
3. Prove that every cauchy sequence is convergent.
4. Prove that, if f and g are continuous then $f + g$ and fg are continuous.
5. Mention properties of continuous function.
6. Show that $f(x) = x^2$ is uniformly continuous on the interval [0,2].
7. Prove that every differentiable function is continuous.
8. Discuss the applicability of Rolle's Theorem for $f(x) = |x|$ in the interval $[-1,1]$.
9. Find the Taylor's series expansion of $f(x) = e^x$.
10. Explain Riemann Integration.
11. Prove that every continuous function is Riemann Integrable.
12. If $f \in R[a, b]$ then prove that $|f| \in R[a, b]$.

PART – B

Note: Answer all the questions. **(4 x 12 = 48 Marks)**

13. (a) (i) For a sequence $\{S_n\}$ of positive real numbers, prove that $lim S_n = +\infty$ if and only If $\lim\left(\frac{1}{S_n}\right) = 0$.

(ii) If $\{S_n\}$ converges to a positive real number 's' and $\{t_n\}$ is any sequence then prove that $\lim sup s_n t_n = s \lim\sup t_n$.

(OR)

(b) (i) Show that the series $\sum \frac{n}{3^n}$ is convergent.

(ii) State and prove Alternating series theorem.

14. (a) If f is continuous on $[a, b]$, then prove that it is bounded and attains it's supremum and infimum.

(OR)

(b) Prove that if f is continuous on $[a, b]$ if and only if it is uniformly continuous on $[a, b]$.

15. (a) State and prove Generalized Mean value theorem.

(OR)

(b) State and prove Taylor's theorem.

16. (a) Prove that a bounded function f on $[a, b]$ is integrable if and only if for each $\epsilon > 0$ there exists a partition p of $f[a, b]$ such that $U(f, p) - L(f, p) < \epsilon$.

(OR)

(b) State and Prove Fundamental theorem of calculus-I

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Code No. 6315

FACULTY OF SCIENCE

B.Sc./B.A. (CBCS) III Semester Examination, March 2022

Subject: Mathematics

Paper-III: Real Analysis

Time: 3 Hours **Max. Marks: 80**

PART – A

Note: Answer any eight questions. **(8 x 4 = 32 Marks)**

1. Define limit of sequence and evaluate the limit of $S_n = \frac{2n+3}{3n+4}$.
2. Prove that convergent sequences are bounded.
3. Find the sub sequential limits of $S_n = \sin\frac{n\pi}{3}$.
4. If f is continuous at x_0 and g is continuous at $f(x_0)$, then prove that the composite function $g_0 f$ is also continuous at x_0.
5. Suppose g is strictly increasing function on an interval J such that $g(J)$ is an interval. Then prove that g is continuous on J.
6. Show that $f(x) = \frac{1}{x^2}$ is uniformly continuous on $[a, \infty)$ where $a > 0$.
7. Prove that every differentiable function is continuous.
8. Discuss the applicability of Rolle's theorem for $f(x) = |x|$ on $[-1,2]$.
9. Find the Taylor series for $f(x) = Sinx$ about zero.
10. If f is a bounded function on $[a, b]$ then prove that $L(f) \leq U(f)$.
11. Prove that every continuous function is Riemann Integrable.
12. If f is Integrable on $[a, b]$, then prove that |f| if Integrable on $[a, b]$.

PART – B

Note: Answer any four questions. **(4 x 12 = 48 Marks)**

13. Prove that a sequence is a convergent sequence if and only if it is a Cauchy sequence.

14. State and prove Alternating series theorem.

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15. Suppose f is a continuous real valued function on a closed interval $[a,b]$. Then prove that f is a bounded function. Also prove that f assumes its maximum and minimum values on $[a,b]$.

16. Prove that a real valued function f on (a,b) is uniformly continuous on (a,b) if and only if it can be extended to a continuous function $\bar{f}$ on $[a,b]$

17. State and prove Rolle's theorem

18. State and prove Taylor's theorem.

19. Prove that a bounded function of on $[a,b]$ is Integrable if and only if for each $\in>0$ there exists a partition P of $[a,b]$ such that $U(f,p) - L(f,p) < \in$.

20. State and prove Fundamental Theorem of calculus – I.

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02-12-2021

Code No. 18073/O/BL

FACULTY OF SCIENCE

B.A. / B.Sc. III Semester (CBCS) Examination, November / December 2021

Subject: Mathematics

Paper – III : Real Analysis

Time: 2 Hours **Max. Marks: 80**

PART – A

Note: Answer any four questions. **(4 x 5 = 20 Marks)**

1 Define limit of sequence and evaluate the limit of $S_n = \frac{3n+1}{4n-1}$.

2 Prove that all bounded monotone sequences converge.

3 Prove that every sequence $[S_n]$ has monotonic subsequence.

4 Show that the series $\sum \frac{n}{3^n}$ is convergent.

5 Find the interval of convergence for the series $\sum_{n=0}^{\infty} \frac{x^n}{n^2}$.

6 Show that $f_n(x) = \frac{x}{1+nx^2}, x \in R$ converges uniformly on R.

7 Give an example of a function which is not Riemann Integrable.

8 If f and g are integrable on $[a, b]$ and if $f(x) \leq g(x)$ for $x \in [a, b]$ then prove that $\int_a^b f \leq \int_a^b g$.

PART – B

Note: Answer any three questions. **(3 x 20 = 60 Marks)**

9 (i) Prove that all bounded monotone sequences converge.

(ii) Given a sequence $S_n = \frac{S_{n-1}^2 + 5}{2S_{n-1}}$ for $n \geq 2$ and $S_1 = 5$ then show that $\sqrt{5} < S_{n+1} < S_n \leq 5$ for $n \geq 1$.

10 (i) Prove that convergent sequences are Cauchy sequences.

(ii) Prove that Cauchy sequences are bounded.

11 Suppose $\{S_n\}$ is any sequence of non-zero reals, then prove that

$$Lim \inf \left|\frac{S_{n+1}}{S_n}\right| \leq Lim \inf |S_n|^{\frac{1}{n}} \leq Lim \sup |S_n|^{\frac{1}{n}} \leq Lim \sup \left|\frac{S_{n+1}}{S_n}\right|.$$

12 State and prove alternating series theorem.

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13 (i) Show that $f_n(x) = x^n$, $x \in [0,1]$ converges point wise but not uniformly on $[0, 1]$.
(ii) Suppose $[f_n]$ is a sequence of functions defined and uniformly Cauchy on a set $S \subseteq R$. Then prove that there exists a function f on S such that $f_n \to f$ uniformly on S.

14 (i) Prove that the uniform limit of continuous function is continuous.
(ii) State and prove Weierstrass M-Test.

15 (i) Prove that a bounded function f on $[a, b]$ is integrable if and only if for each $\in > 0$ there exists partition P such that $U(f,p) - L(f,P) < \in$.
(ii) If f and g are integrable on $[a, b]$ then prove that $f + g$ is integrable and $\int_a^b f + g = \int_a^b f + \int_x^b g$.

16 (i) Prove that every continuous function f on $[a, b]$ is integrable.
(ii) State and prove fundamental theorem of Calculus-I.

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OU - 1110 OU - 1110

Code No. 18073

FACULTY OF SCIENCE

BA / B.Sc. III Semester (CBCS) Examination, July 2021

Subject: Mathematics

Paper: III - Real Analysis (DSC)

Time: 2 Hours | Max. Marks: 80

Note: Missing data, if any, may be suitably assumed

PART – A

Note: Answer any five questions. (5 x 4 = 20 Marks)

1 Show that every cauchy sequence is bounded.

2 Determine the nature of the series $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2}$.

3 Let $f: R \to R$ be a function defined as follows

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0 & , x = 0 \end{cases}$$

Then show that f is continuous at $x = 0$.

4 Let f and g be any two continuous functions defined on $[a,b]$ such that $f(a) \geq g(a)$ and $f(b) \leq g(b)$. Then show that $f(x_0) = g(x_0)$ for at least one $x_0 \in [a,b]$.

5 Let $f: R \to R$ be a function. Suppose that $|f(x) - f(y)| \leq (x-y)^2$ for all $x, y \in R$. Then show that f is a constant function.

6 Find the Taylor series of the function $f(x) = \log(1+x)$ for $-1 < x < \infty$.

7 Let $f:[a,b] \to R$ be a bounded function. If P, Q are any two partitions of $[a,b]$ such that $P \subseteq Q$ then show that $L(f,P) \leq L(f,Q)$.

8 Let $f:[a,b] \to R$ be a Riemann integrable function and $c \in R$. Then show that cf is integrable and $\int_a^b cf = c\int_a^b f$.

9 Suppose $t_1 = 1$ and $t_{n+1} = \left(1 - \frac{1}{4n^2}\right) t_n$ for $n \geq 1$. Then evaluate $\lim t_n$ if it exists.

10 Determine whether the series $\sum_{n=2}^{\infty} \frac{\log n}{n^2}$ is convergent.

11 Show that $f_n(x) = \sum \frac{x^n}{1+x^n}$ converges for $x \in [0,1)$.

12 Prove that if f is integrable on [a,b] then f^2 is also integrable on [a,b].

PART – B

(3 x 20 = 60 Marks)

Note: Answer any three questions.

13 (i) Show that a sequence is a convergent sequence if and only if it is a Cauchy sequence.

(ii) Evaluate $\lim_{n\to\infty} \frac{1}{n}(n!)^{\frac{1}{n}}$.

14 (i) State and prove the Root test.

(ii) Evaluate $\sum_{n=2}^{\infty}\left(-\frac{1}{3}\right)^{n}$.

15 State and prove intermediate value theorem.

16 (i) Show that the function $f(x)=\frac{1}{x^2}$ is uniformly continuous on $[a,\infty)$ where $a>0,\ a\in R$.

(ii) If $f:[a,b]\to R$ is continuous, then show that f is uniformly continuous.

17 (i) State and prove Rolle's Theorem.

(ii) Show that $ex \le e^x$ for all $x \in R$.

18 (i) Show that the function $f(x)=\frac{x}{\sin x}$ is a strictly increasing function on $\left(0,\frac{\pi}{2}\right)$.

(ii) Evaluate $\lim_{x\to 0}\left(\frac{1}{e^x-1}-\frac{1}{x}\right)$.

19 Let f be a bounded function defined on $[a,b]$. If $a<c<b$ and f is integrable on $[a,c]$ and on $[c,b]$, then show that f is integrable on $[a,b]$ and $\int_a^b f = \int_a^c f + \int_c^b f$.

20 Let $g:[a,b]\to R$ be a continuous function on $[a,b]$ and differentiable on (a,b). If g' is integrable on $[a,b]$ then show that $\int_a^b g' = g(b)-g(a)$.

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Code No. 18073/N/BL

FACULTY OF SCIENCE

B.A. / B.Sc. III Semester (CBCS) Examination, November / December 2021

Subject: Mathematics
Paper: III Real Analysis

Time: 2 Hours **Max. Marks: 80**

PART – A

Note: Answer any four questions. **(4 x 5 = 20 Marks)**

1 Show that the sequence $\{(-1)^n\}$ does not converges.

2 Let $a_n \geq 0 \forall \ n \geq 1$. If $\sum_{n=1}^{\infty} a_n$ is convergent then show that $\sum_{n=1}^{\infty} a_n^2$ is convergent.

3 Prove that $x2^x = 1$ for some $x \in (0,1)$.

4 Show that the function f: **R**→**R** defined by $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is discontinuous at $x = 0$.

5 Show that $|\cos x - \cos y| \leq |x - y|$ *for all* $x, y \in$ **R**

6 Evaluate $\lim_{x \to 0} \frac{1 - \cos 2x - 2x^2}{x^4}$.

7 If $f : [a,b] \to$ **R** is a bounded function, then show that $L(f) \leq U(f)$.

8 If $f : [a,b] \to$ **R** is a monotonic function, then show that f is Riemann integrable.

PART – B

Note: Answer any three questions. **(3 x 20 = 60 Marks)**

9 (i) Show that every convergent sequence is bounded.
(ii) Show that an increasing bounded sequence is convergent.

10 (i) Determine the nature of the series $\sum_{n=1}^{\infty} \frac{(100)^n}{n!}$.

(ii) Show that the series $\sum_{n=1}^{\infty} 2^{(-1)^n - n}$ is convergent.

11 If $f : [a,b] \to$ **R** is a continuous function, then show that f is bounded. Further show that there exists $x_0, y_0 \in [a,b]$ such that $f(x_0) \leq f(x) \leq f(y_0)$ for all $x \in [a,b]$.

12 (i) Show that the function defined by $f(x) = \frac{1}{x^2}$ for $x \in (0,1)$, is not uniformly continuous on $(0,1)$.

(ii) Show that the function f : **R**→**R** defined by $f(x) = 3x + 11$, is uniformly continuous on **R**.

13 (i) State and prove the Mean value theorem.
(ii) Let f be a differentiable function on (a,b) such that $f'(x)=0$ for all $x\in(a,b)$.
Then show that f is a constant function on (a,b).

14 Let f be defined on (a,b) where $a<0<b$ and suppose that n^{th} derivative $f^{(n)}$ exists on (a,b). Then for each non zero $x\in(a,b)$, show that there is some $y\in(0,x)$ such that $R_n(x)=\frac{f^{(n)}(y)}{n!}x^n$.

15 Show that a bounded function $f:[a,b]\to\mathbf{R}$ is integrable if and only if for each $\varepsilon>0$ there exists a partition P of $[a,b]$ such that $U(f,P)-L(f,P)<\varepsilon$.

16 Let $f:[a,b]\to\mathbf{R}$ be an integrable function. For $x\in[a,b]$, let $F(x)=\int_a^x f(t)\,dt$. Then show that F is continuous on $[a,b]$. Further if f is continuous at $x_0\in(a,b)$, then show that F is differentiable at x_0 and $F'(x_0)=f(x_0)$.

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OU - 1110 OU - 1110

Code No. 8073 / E

FACULTY OF SCIENCE

B.Sc. III-Semester (CBCS) Examination, October / November 2020

Subject : Mathematics (Real Analysis)

Paper – III (DSC)

Time : 2 Hours **Max. Marks: 80**

PART – A (4 x 5 = 20 Marks)

Note : Answer any four questions.

1 Find $\lim_{n\to\infty} s_n$, where $s_n = \sqrt{n^2+1} - n$.

2 Prove that every Cauchy sequence is bounded.

3 Find the set of subsequential limits of the sequence $\{a_n\}$, where $a_n = n(1 + (-1)^n)$.

4 If a series Σa_n converges, prove that $\lim a_n = 0$.

5 Find the interval of convergence of the series $\sum \frac{x^n}{n}$.

6 Define uniform convergence of a sequence of functions.

7 Prove that every monotonic function on [a, b] is integrable.

8 Show that $\left| \int_{-2\pi}^{2\pi} x^2 \sin^8(e^x)\, dx \right| \le \frac{16\pi^3}{3}$.

PART – B (3 x 20 = 60 Marks)

Note: Answer any three questions.

9 Let $\langle s_n \rangle$ be a sequence of non-negative real numbers and suppose that $s = \lim s_n$. Then prove that $\lim \sqrt{s_n} = \sqrt{s}$.

10 Prove that :

(i) $\lim_{n\to\infty}\left(n^{-\frac{1}{p}}\right) = 0$ for $p > 0$ (ii) $\lim_{n\to\infty} a^n = 0$ if $|a| < 1$.

11 State and prove Ratio-test.

12 If $a_1 \ge a_2 \ge \ldots \ge a_n \ge \ldots \ge 0$ and $\lim_{n\to\infty} a_n = 0$ then prove that $\sum (-1)^{n+1} a_n$ converges.

13 Prove that $(f_n(x))$, , where $f_n(x) = \frac{x}{1+nx^2}$, $x \in R$, converges uniformly on **R**.

14 Show that if the series Σg_n converges uniformly on a set S, then $\lim_{n\to\infty} \sup\{|g_n(x)| : x \in S\} = 0$.

15 Let f and g be integrable on [a, b]. Prove that f + g is integrable and $\int_a^b f + g = \int_a^b f + \int_a^b g$.

16 State and prove intermediate value theorem for integrals.

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